

The funny people of Paris  
Think it is 2:46 PM now.

ATMs

Exponentials are like bank machines in dreamland - you can withdraw as much cash as you wish, you can keep or throw or spend the cash, yet the ATM remains completely unchanged.

Mathematical formulation: With  $V$  the v.s w/ basis  $\{\$5, \$10, \dots, \boxed{\$5}, \boxed{\$20}\}$ , with  $\Delta: S(V) \rightarrow S(V)$  the "split the cash coproduct", with  $P: S(V) \rightarrow W$  the purchasing map, we have:

$$\begin{array}{ccc}
 \overset{\text{ATM}}{V} & \xrightarrow{\quad} & \overset{\text{ATM}}{V} \otimes \overset{\text{ME}}{V} \xrightarrow{I \otimes P} V \otimes W \\
 e^x & \xrightarrow{\quad} & e^x \otimes P(e^x)
 \end{array}$$

This of course is a generalization of  $(e^x)' = e^x$ , with  $P: \begin{matrix} 1 \rightarrow 0 \\ x \rightarrow 1 \\ x^k \text{ with } k \geq 1 \rightarrow 0 \end{matrix}$ .

So exponentials are vector-space analogs of Hilbert's hotel; they are "infinite reservoirs"; take one (or many) out, and the reservoir remains unchanged.