Knotted Trivalent Graphs, Tetrahedra andAssociators

Trieste Day 4 handout, May 2009 (based on HUJI--001116)

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Modulo the relation(s):

\[
\begin{align*}
\text{Modulo the relation(s):} & \quad \begin{array}{c}
\text{(101)}
\end{array} = \begin{array}{c}
\text{(110)}
\end{array} \\
\text{Claim.} \text{ With } \Phi := Z(A), \text{ the above relation becomes equivalent to the Drinfel'd pentagon of the theory of quasi Hopf algebras.}
\end{align*}
\]

Proof.

\[
\begin{align*}
\Phi \in A(\Delta)
\end{align*}
\]

I hope to work on the U case after completing the \( W \)-warmup.

Extend to Knotted Trivalent Graphs (K TG’s):

\[
\begin{align*}
\text{Extend to Knotted Trivalent Graphs (K TG’s):} & \quad \begin{array}{c}
\text{K TG’s}
\end{array} \\
\text{Need a new relation:} & \quad \begin{array}{c}
\text{Relation}
\end{array} \\
\text{Easy, powerful moves:} & \quad \begin{array}{c}
\text{Movements}
\end{array} \\
\text{Using moves, K TG is generated by ribbon twists and the tetrahedron} & \quad \begin{array}{c}
\text{Tetrahedron}
\end{array}
\end{align*}
\]

What are associators good for? 0. Conformal expansions:
1. Quantum group and the Kohno-Drinfel’d theorem, also
   heavily used in knot theory.
2. Etingof-Kazhdan quantization of Lie algebras.
3. Turaev’s proof of Kontsevich’s deformation quantization.
4. Alekseev-Torossian’s study of Witten–
   Verlinde.
5. Maybe more?
See more at http://www.math.toronto.edu/~drorbn/Talks/Trieste-0905