A Kac–Moody algebra is given by the following:

1. An $n$ by $n$ generalized Cartan matrix $C = (c_{ij})$ of rank $r$.
2. A vector space $\mathfrak{h}$ over the complex numbers of dimension $2n - r$.
3. A set of $n$ linearly independent elements $\alpha_i$ of $\mathfrak{h}$ and a set of $n$ linearly independent elements $\alpha^*_i$ of the dual space, such that $\alpha^*_i(\alpha_j) = c_{ij}$. The $\alpha_i$ are known as coroots, while the $\alpha^*_i$ are known as roots.

The Kac–Moody algebra is the Lie algebra $\mathfrak{g}$ defined by generators $e_i$ and $f_i$ and the elements of $\mathfrak{h}$ and relations:

- $[e_i, f_j] = \alpha_i \delta_{ij}$
- $[e_i, f_j] = 0$ for $i \neq j$
- $[e_i, x] = \alpha^*_i(x) e_i$ for $x \in \mathfrak{h}$
- $[f_i, x] = -\alpha^*_i(x) f_i$ for $x \in \mathfrak{h}$
- $[x, x'] = 0$ for $x, x' \in \mathfrak{h}$
- $\text{ad}(e_i)^{1-c_{ii}}(e_i) = 0$
- $\text{ad}(f_i)^{1-c_{ii}}(f_i) = 0$

[$e_i, e_j$], $[f_i, f_j]$ = $[e_i, [e_j, f_i]] + [f_i, [e_j, e_i]]$

= $[\alpha_i, e_j], f_i] + [f_i, [e_i, \alpha_j]]$

= $-\alpha^*_j(\alpha_i) e_i, f_j] + [f_i, \alpha^*_j(\alpha_i)] e_i$

= $-\alpha^*_j(\alpha_i) \alpha_j - \alpha^*_i(\alpha_j) \alpha_i$

= $-C_{ij} \alpha_i \alpha_j - C_{ij} \alpha_i$