A parity $p$ on a “knot theory” (a theory of words in the sense of Turaev) is an assignment of a parity in $\mathbb{Z}/2$ for every knot diagram, so that

1. $p$ is invariant relative to the “identity of xing” partial connection [so it is local and has a $\not\exists\not\exists$ property and $R3$].

2. The sum of the parities of the xings involved in an $R1$, $R2$, or $R3$ move is even.

**Examples**

1. The number of chords (mod 2) intersecting a given chord, for $v$-knots.

2. For links, the parity of the number of components involved in a given xing.

**Question** Is there a non-trivial parity for honest 1-component $v$-knots?

The “odd & irreducible” is minimal in a strong sense follows from

$$\text{prop} \sum_{\text{mod}2} (\text{1-component Kauffman smoothings of even xing}) / \text{moves}$$

is invariant also under $R3$. 


Another corollary: Non-trivial free knots exist

Free knots = CA \langle \star \rangle / R1, R2, R3 & \star

Question: Classify free knots.