A. Suppose \( (\text{gr} \ Z^0) \circ \tau = \text{Id} \). Then \( \tau \) and \( \text{gr} \ Z^0 \) are isomorphisms.

B. Identifying \( A = \text{gr} \ K \) and renaming \( Z^0 \) to \( Z \), \( Z \) is an expansion for \( K \).

Why was I so reluctant back when I wrote the Tianjin/Hanoi handouts?

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**Precise formulation** Let \( K \) be filtered.

Assume \( A \) is graded, \( \tau : A \to K \) is onto, and we have a filtered \( \tau : K \to A \) st. \( \text{gr} \ Z^0 \circ \tau = \text{Id} \).

\( Z \) is an \( A \)-expansion]. Then \( \tau : A \cong \text{gr} \ K \) and \( Z = \tau \circ Z^0 \) in an expansion.