Homomorphic Expansions

The general setup:

\[ \phi : K \to K_0 \rightarrow K_1 \rightarrow K_2 \rightarrow K_3 \rightarrow \ldots \]

\[ \downarrow \quad \downarrow \quad \downarrow \quad z \]

\[ \text{gr } K = K_0 \oplus K_1 \oplus K_2 \oplus K_3 \oplus K_4 \oplus \ldots \]

An expansion is a filtration-respecting map \( K \to \text{gr } K \) that "covers" the identity map \( \text{gr } K \to \text{gr } K \).

A homomorphic expansion is an expansion that respects all relevant "extra" operations.

A concrete example:

\[ \phi K = \mathcal{O} \]

The set of all b/w 2D projections of reality

\[ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \text{an "expansion" } Z \text{ is a choice of a "progressive scan" algorithm.} \]

\[ K/K_0 \oplus K/K_1 \oplus K/K_2 \oplus K/K_3 \oplus K/K_4 \oplus \ldots \]

added May 5, 2009: I was a bit confused .... See Talks/Tristle.

Our case:

\[ \text{Knot Theory} \quad \text{Topology} \]

Finding \( Z \) is solving finitely many thousands of equations in finitely many unknowns

\[ \begin{align*}
\text{finite combinatorics:} & \quad \text{bounded, complex, diagrams mod simple relations} \\
\text{Low Algebra:} & \quad \text{pictures represent formulas}
\end{align*} \]