 Characters of finite Chevalley groups and categorification

W. Finkelberg, Ostrik.

G: Finite group

A representation \( p : G \to \text{GL}(V, q) \) (homomorphism)

assume \( p \) is irreducible.

The character \( \chi_p : G \to \mathbb{C} : \chi_p(g) = \text{Tr}(p(g)) \)

Questions 1. Describe \( \text{Irr}(G) \)

2. Compute all \( \chi_p \) s.

Concentrate on \( G \) like \( \text{GL}_n(\mathbb{F}_q) \); in general, \( G = G(\mathbb{F}_q) \) where \( G \) is an algebraic group over \( \mathbb{F}_q \).

(Solved by Lusztig, sometimes with brute force)

Goal: A more conceptual approach to this theory.

characters are functions \( (G = X)(\mathbb{F}_q) \to \mathbb{C} \)

Grothendieck's idea: such can come from:

\[
X(\mathbb{F}_q) = X(\mathbb{F}_q)^{F \text{red}}
\]

\[
\downarrow
\]

\[
M^F \quad \text{a manifold } M^F/\text{an automorphism } F
\]

A function could come from an equivariant sheaf on \( F \)

\[ F_x \quad \text{a sheaf } \quad \text{and } \text{isomorphism } \quad F_x \cong F_{F^x} \]

\( \Rightarrow \) get traces at the fixed points of \( F \).

Really, we will violate "sheaves" with "complexes"
Really we will replace "sheaves" with "complexes of sheaves". More specifically "purposes sheaves".

Induced representations: Given a representation $\rho$ of $H \leq G$: \[(\rho : H \to GL(V))\]

\[\text{Ind}_H^G(\rho) = \{F : G \to V : F(gh) = \rho(h^{-1})F(g)\}\]

Suppose \[G \supset B \longrightarrow T\]

"Borel" "Cartan" (Abelian)

If $\chi \in T^V$ set \[i_\chi = \text{Ind}_B^G(\chi)\]

For most $\chi$ this is irreducible, though consider $\chi = 1$ \[\Rightarrow i_\chi = \mathbb{C}[G/B]\]

Set \[\mathcal{P}_i := \{p \mid p \supset i_\chi\} \subset p \supset 0\]

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