Question: Is there a basis conjugating automorphism $F_2$ of $F(a,b)$ s.t. $F_2(a^2b^2) = (ab)^2$?

And in general, $F_n$ s.t. 

$F_n(a^n b^n) = (a b)^n$ ?

The interest comes from Trotter's formula,

$e^{\alpha \gamma} = \lim_{n \to \infty} (e^{\alpha / n} e^{\gamma / n})^n$

By BCH, $F = \lim_{n \to \infty} (e^{\alpha / n} e^{\gamma / n})^n$

By taking $a = e^{\alpha / n}, b = e^{\gamma / n}$ this is

$= F_n(e^{\alpha \gamma})$

So it look like $F = \lim_{n \to \infty} F_n^{-1}$

Thus knowing the $F_n$'s will give us $F$.

Are $a b a b$ and $a a b b$ conjugates? No, by are different cyclic words.