For reasons unknown, this notebook page got messed up. A backup PDF is in the parent directory.

KAL-090128: Lie bialgebras, gl(N), framing v-knots
January 28 09
9:53 AM

Results with Louis:

<table>
<thead>
<tr>
<th>n</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>dim $\mathfrak{gl}_n$</td>
<td>1</td>
<td>2</td>
<td>7</td>
<td>27</td>
<td>139</td>
</tr>
<tr>
<td>dim im $T_{gl(n)}$</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>22</td>
<td>hgs</td>
</tr>
<tr>
<td>dim im $T_{gl(n)}$</td>
<td>1</td>
<td>2</td>
<td>7</td>
<td>27</td>
<td>118</td>
</tr>
</tbody>
</table>

Something must be right and something is going on. Are there really two $gl(N)$ constructions? Which of them is "right"? [Some ramon thoughts on the matter are below]

* Reminder: $\mathfrak{h}/6T$

* Reminder: Lie bialgebras and arrow weight systems

* Reminder: $\mathfrak{l_r} \oplus U = gl(n) \oplus H$

\[ [e^{ij}_{kl}, e^{ij}_{kl}] = \frac{1}{2} (e^{ij}_{kl} - e^{ij}_{lk}) \]

with $h_{ij} = e_{ij} = \frac{1}{2} (e^{ij}_{kl} + e^{ij}_{lk})$

* For $gl(N)$

That is some specific $v \in gl(N) \otimes \mathfrak{g}(N)$ solves the T/YBE $[6T]$ in $U(gl(N))^{\otimes 3}$

* \[
\begin{array}{ccc}
\tilde{A}(\gamma) & \xrightarrow{T_{gl(n)}} & U(gl(n)) \\
\downarrow & & \downarrow \\
\tilde{A}(\gamma_{\mathfrak{gl}(n)}) & \xrightarrow{T_{gl(n)} \otimes H} & U(gl(n) \otimes H) = U(gl(n)) \otimes S(H)
\end{array}
\]
Results with Louis:

\[
\begin{array}{cccccc}
\dim \mathfrak{g}\ell_n & 0 & 1 & 2 & 3 & 4 \\
\dim \mathfrak{g}\ell_{n+1} & 1 & 2 & 7 & 27 & 139 \\
\dim \mathfrak{t}_{g\ell(n)} & 1 & 2 & 6 & 22 & \text{less} \\
\dim \mathfrak{t}_{g\ell(n+1)} & 1 & 2 & 7 & 27 & 318
\end{array}
\]

Some thing must be right and something is going on. Are there really two \(g\ell(n)\) constructions? Which of them is "right"?

\[\text{[some random thoughts on the matter are below]}\]

* Reminder: \(\mathfrak{t}_{g\ell(n^2)}\)

* Reminder: Lie bialgebras and arrow weight systems

* Reminder:

\[
\begin{aligned}
\mathfrak{g}\ell(n) &\cong \mathfrak{g}\ell(n) \oplus \mathbb{H} \\
\{e_{ij}\}_{i\neq j} &\oplus \{e_{ij}^-\}_{i\neq j} \oplus \{e_{ii}\} \\
h_{ij} = e_{ij}^- = \frac{1}{2}(e_{ii}^- + e_{ij}^-)
\end{aligned}
\]

* \(6T\) For \(g\ell(n)\)

(That is, some specific \(v \in g\ell(N) \otimes g\ell(N)\) solves the YBE [6T] in \(U(g\ell(n))^{\otimes 3}\))

* \(\mathfrak{t}_{g\ell(n)} \rightarrow U(g\ell(n)) \rightarrow \mathfrak{t}_{g\ell(n)}\)

* In Lie world, \(\mathfrak{t}_{g\ell(n^2)} \rightarrow \mathfrak{t}_{g\ell(n^2)^+}\)

* In topology, \(\mathfrak{t}_{g\ell(n^2)} \rightarrow \mathfrak{t}_{g\ell(n^2)^+}\) represents a "homology class"
* In topology, $\tilde{A}(\mathbb{R}^n \setminus X)$ represents a "homology class" in the complement of $X$.

* Example: $\tilde{A}(\mathbb{R}^n \setminus \mathbb{R}^n \setminus \{0\}) \cong \tilde{A}(\mathbb{R}^n)$ as vector spaces, though I'm not sure how compatible this is with $\tilde{A}(\mathbb{R}^n)$.

* Remember that even on framed $\nu$-knots, $\mathcal{K}$ is not the only relation.

This all suggests that it may be that the right objects to study are $\nu$-framed $\nu$-knots—these would be $\nu$-knots with a $\nu$-homology class in their complement.

I don't expect this to be the truth, only a step in the right direction