A formal picture of QFT: Input:

\[ S(\phi) = \int M L(\phi) \] typically \[ L(\phi) = \int \frac{\phi(\Delta + m^2)\phi}{m + \text{perturbations}} \]

Output: The "statistical system" of a random field \( \phi \in C^\infty(M, \mathbb{R}) \) with the measure \[ e^{iS(\phi)/\hbar} \, d\mu \text{Lebesgue or } e^{-S(\phi)/\hbar} \, d\mu \]

That is, \( \mathcal{O}(\phi) = \int M e^{i\phi} \) if we want

\[ \mathbb{E}(\mathcal{O}_1(\phi_1), \mathcal{O}_2(\phi_2), ..., \mathcal{O}_n(\phi_n)) \] Typically a distribution on \( M^n \)

Minkos' theorem says that the free Euclidean measure for \( \phi(0 + m^2)\phi \) actually does exist, and is supported on distributions.

**Def** A formal measure on \( D(M) \) is a continuous linear map

\[ \mathcal{T} : C^\infty(M^n, S^1) \to \mathbb{R}[T] \]

Then there is a bijection between

1. Lagrangians \( L(\phi) = \sum \lambda_i L_i(\phi) \)

   where \( L_0(\phi) = \phi(0 + m^2)\phi + \text{cubic + higher} \)

   up to the addition of a total derivative
2. Formal measures satisfying a "locality" property: