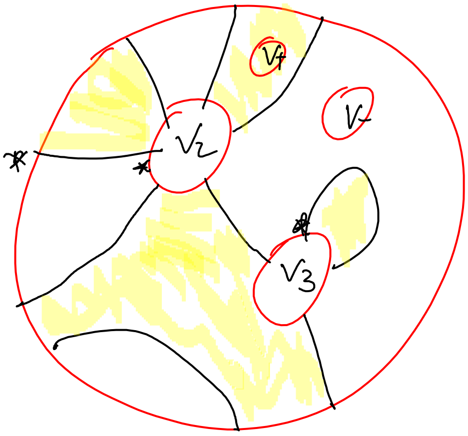
Shaded Planar Algebra: Spaces: $\{V_i\}_{i=+,-,1,2,3,\ldots}$

operations:

stars are always in unshaded regions.



$: V_+ \otimes V_- \otimes V_2 \otimes V_3 \to V_4$

with the obvious associativity....

<u>Example</u> Temperley-Lieb: $\dim TL_n = \frac{1}{n+1}\binom{2n}{n}$

$$\bigcirc \longrightarrow \delta$$

<u>Example</u> K_n: Tangle diagrams with $2n$ endpoints.

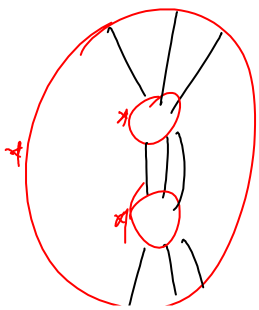
$= PA\langle \text{crossings} \rangle / R123$, moving the star.

A subfactor planar algebra:

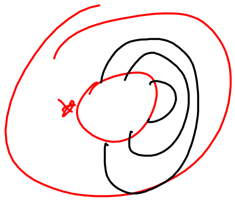
1. $\dim P_+ = 1 = \dim P_-$
2. Involution $*$ on P_i compatible with reflection of ops.
3. Bilinear form: $a, b \in V_k$

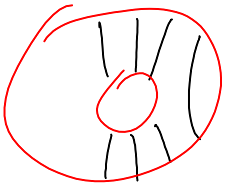
$\langle a, b\rangle := \operatorname{trace}(b^* a)$

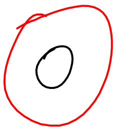
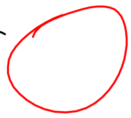
needs to be positive definite.



$: V_k \otimes V_k \longrightarrow V_k$ "The product"

 $: V_k \longrightarrow V_k$ "The Trace"

 $: V_k \longrightarrow V_{k+1}$ "inclusion"

Positivity implies  $= \delta$  $\delta > 0$

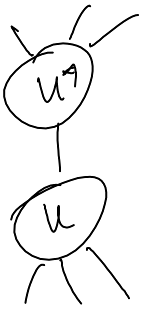
In T-L, $\delta \in \{2\cos\frac{\pi}{n} : n \geq 3\} \cup [2,\infty)$

Fact If P is a subf. P.A then $TL(\delta) \hookrightarrow P$.

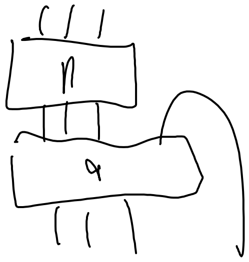
The principal graph:

Vertices: $\{$min. projections$\}/\cong$

where $p \cong q$ when $\exists u$ s.t.

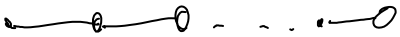
 $= p$  $= q$

Edges whenever



$\neq 0$

For TL at $\delta = 2\cos\frac{\pi}{n}$,

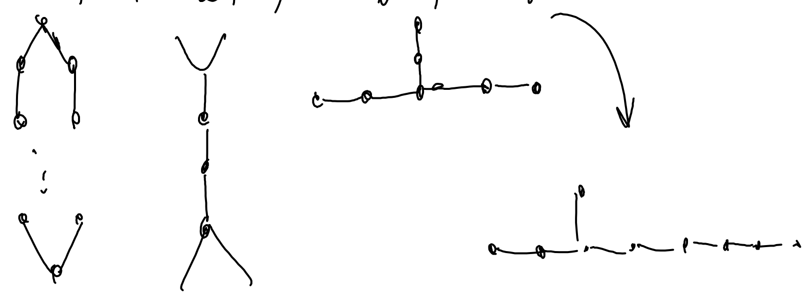
$PG(TL_\delta) =$  A_{n-1}

$\delta \geq 2 \to A_\infty =$ 

Q Which finite graphs are possible?

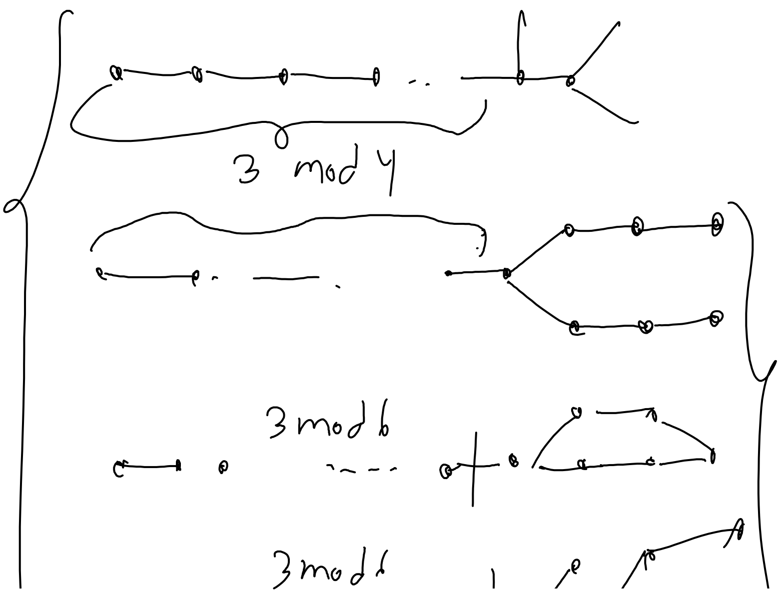
For $\delta < 2$, A_n, D_{2n}, E_6, E_8 (no E_7!)

$\delta = 2$ $\tilde{A}_n$, $\tilde{D}_{2n}$, $\tilde{E}_6$, $\tilde{E}_8$



Haagerup: $2 < \delta < \sqrt{3+\sqrt{3}} \approx 2.175$

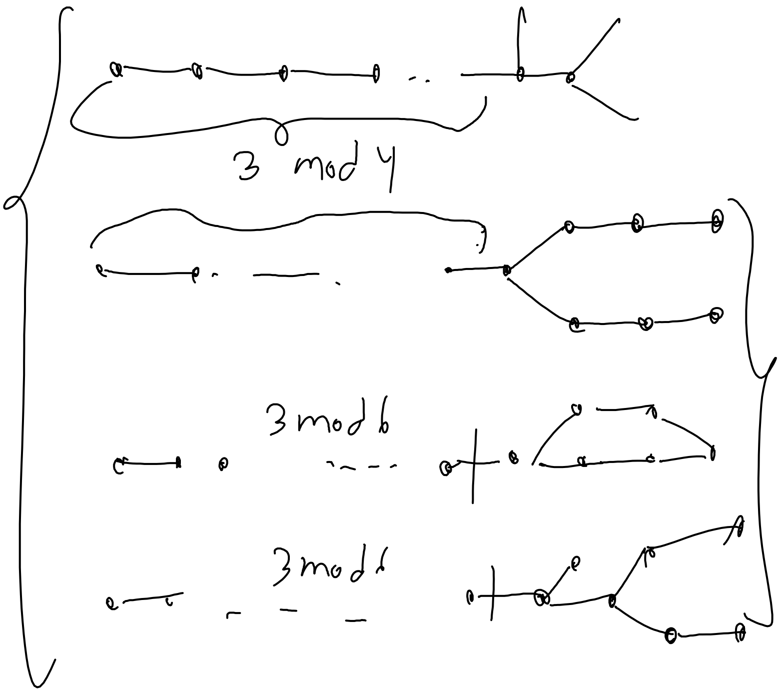
at most



n = 3 done
n = 7 ???

ruled out

at most

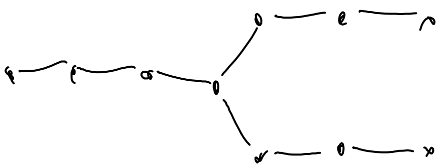


ruled out

n = 3 done
n = 7 ???

+ 2 more (constructed)

Peters' thesis;



Presentation in handout.

$r = \sqrt{\frac{5+\sqrt{13}}{2}} =$

Handout at http://math.berkeley.edu/~eep/HaagerupHandoutToronto2008.pdf