The issue at last? No.
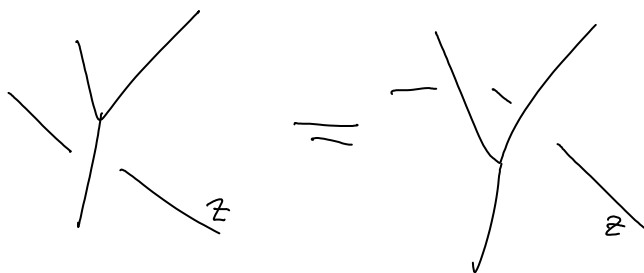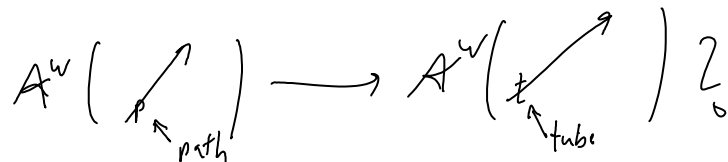
as it stands, only the "z is a homotopy 'strand'" version of this identity is equivalent to a tdr identity.

Is there a map

$$A^w \left(\overset{\rho}{\underset{\text{path}}{\nearrow}} \right) \longrightarrow A^u \left(\overset{q}{\underset{\text{tube}}{\nearrow}} \right) \ ?$$

Yes, and it is ok to verify identities where they are harder.