The Heisenberg Algebra

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$Q = \hat{x}$, $P = \frac{\partial}{\partial x}$, $[P, Q] = \frac{\partial}{\partial x}\hat{x} - \hat{x}\frac{\partial}{\partial x} = I$

Question What's $\exp(\alpha P + \beta Q)$?

Sol'n Let $f_t = \exp t(\alpha P + \beta Q) f$. Then

$$\frac{\partial}{\partial t} f_t = \left(\alpha \frac{\partial}{\partial x} + \beta x\right) f_t \qquad f_0 = f$$

Alternatively, $(\alpha P + \beta Q)^n = \sum_{k=0}^{n} (\alpha P)^k (\beta Q)^{n-k} \cdots$

$Q Q \ldots P Q Q \ldots P Q \ldots Q$

$(1 + c + \ldots + c^k)($

Say $f_t = e^{tx} f(x+t)$. Then

$$\left(\frac{\partial}{\partial t} f_t\right)(x) = x e^{tx} f(x+t) + e^{tx} f'(x+t)$$

So

$$\left(\frac{\partial}{\partial t} f_t\right)\Big|_{t=0} = x f(x) + \frac{\partial}{\partial x} f(x)$$

Lookup "The Moyal Product".