Injectivity

November-14-08
4:12 PM

<u>Exercise</u> Prove the injectivity of Z^{wB} by showing that the expansion of an automorphism determines the expansion of any image thereof.

$$\begin{tikzcd} \beta \in wB_n \arrow[r, bend left] \arrow[d, "Z"'] & F_n \ni x \arrow[d, "Z"] \\ \mathcal{A}^w_n \arrow[r, bend left] & FA_n \end{tikzcd}$$

commutes?

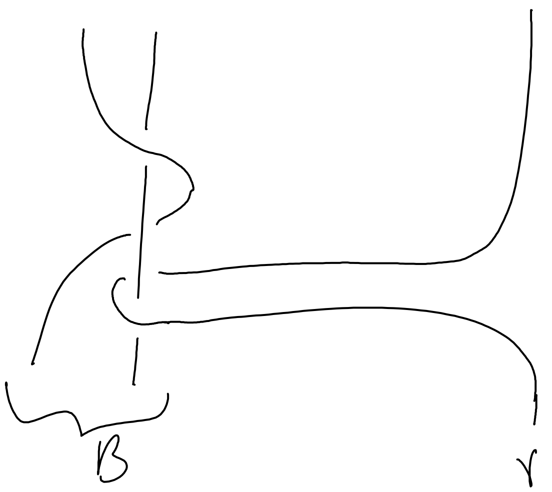
✓

If commutes, we are done:

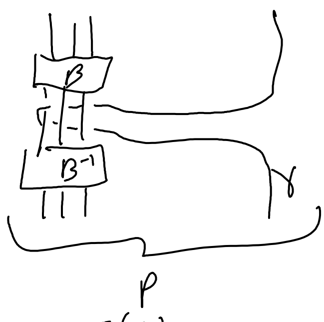
$$Z(\beta x) = Z(\beta) Z(x)$$

<u>Exercise</u> Now do the same for Z^B

✓



$$Z(\beta\gamma) = Z(\beta) \cdot Z(\gamma)$$



$$\tau(Z(\beta\gamma)) = Z(P) = (\tau Z(\gamma))^{Z(\beta)} = \tau\left(Z(\gamma)^{Z(\beta)}\right)$$

but τ is injective, so $Z(\beta\gamma) = Z(\beta) Z(\gamma)$

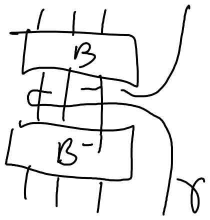
Necessary claims:

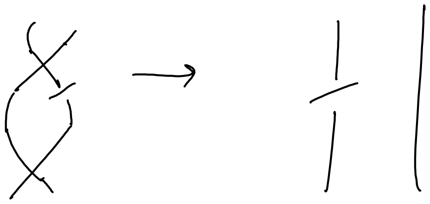
pf'l by coming

$\iota : FA_n \longrightarrow A_{n+1}$ is 1-1.

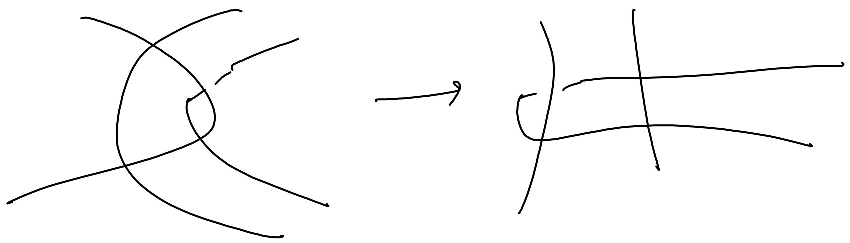
$\iota^w : FA_n \longrightarrow A^w_{n+1}$ is 1-1.

$\Psi : wB_n \longrightarrow \mathrm{Aut}(F_n)$ as conjugation:

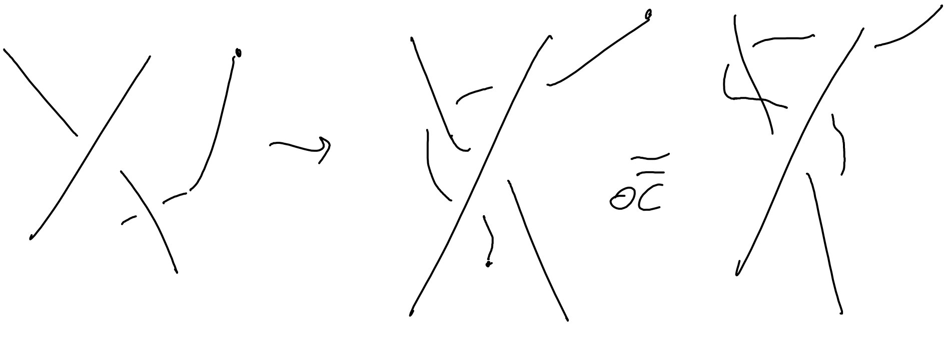


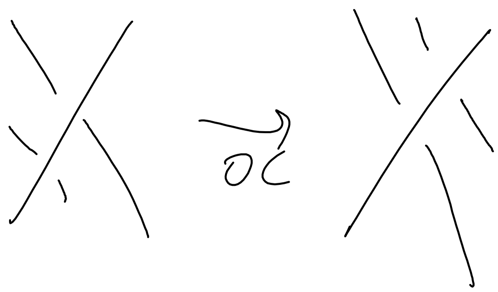


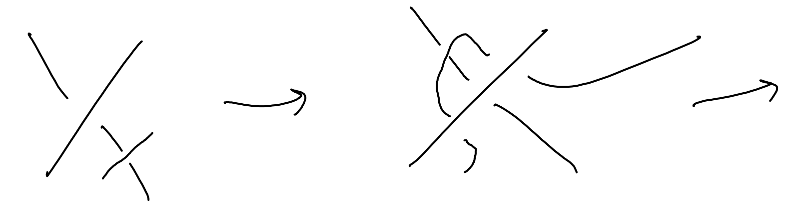
by detour

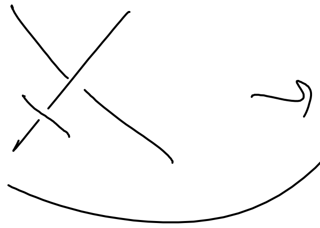


by a detour move.

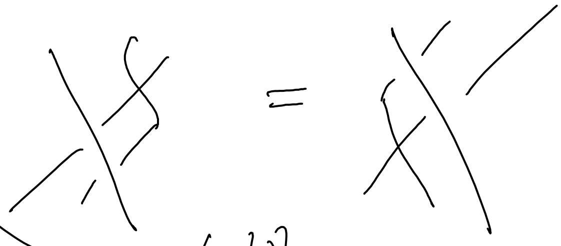








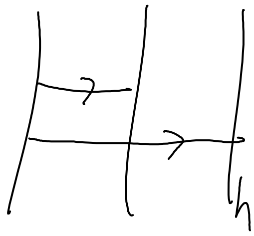
Q in vB, is



Nyet!

Moral:

The relation



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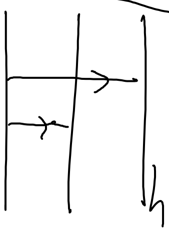

must be imposed for homotopy strand in vKO!

(tails commute, if one of the heads is on the homotopy strand)