$m: g \times g \rightarrow g$ is $x, y \mapsto x + y$

$M: g \times g \rightarrow g$ is $x, y \mapsto \log e^x e^y$

$g \times g \xrightarrow{m} g \xrightarrow{\exp} g \xrightarrow{\exp} G \xrightarrow{m} G$

Claim $w$ to $v$'s,

$M_* = M_*$ on $G \times G$-invariant

Added Dec 9, 2008: An Abstract setup: [Tentative]

$(H \times G, \text{for}) \xrightarrow{\mu} H \xrightarrow{\phi} G \xrightarrow{\alpha^\#} Y$

$\alpha_1^\# \beta: X \rightarrow Y$

When does this happen?

$X \times H \xrightarrow{\rho} X \xrightarrow{\alpha \beta} Y$

$H$-orbits.

$\mathcal{L}_\mu (\mu \times \nu) = \mathcal{L}_\nu (\mu \times \nu)$ for all $\mu, \nu \in \mathcal{M}_1 (X)$

A neighboring question: (The restriction of the above question to a single orbit)

Suppose $(X, \nu)$ is a measure space and $\alpha, \beta: X \rightarrow Y$ are transformations such that $\alpha_* \nu = \beta_* \nu$. Is it reasonable to insist that the equality $\alpha_* \nu = \beta_* \nu$ be derived from
the existence of a commutative triangle
\[ X \xrightarrow{T} X \] in which \( T \) is measure preserving
(i.e., \( T^* \nu = \nu \))

Aside: if \( G \leq X \) then \( G \times G \times X \times X \); the \( G \times G \)-orbits
in \( X \times X \) are products of \( G \) orbits in \( X \).

End Dec 9 additions.

Need \( F : g \times g \to g \times g \) s.t.

\[
\begin{array}{ccc}
g \times g & \xrightarrow{\mu} & g \\
\downarrow{(x_0, x_1))(1 \times F)} & & \downarrow{\text{preserves invariant measure \( \mu \)}} \\
g \times g \times g \times g & \xrightarrow{C \times C} & g \times g \\
\end{array}
\]

where \( \sigma(x, y) = (y, x) \)
and
\[
c(x, y) = e^{-y \cdot x} \cdot e^y = e^{-x} \cdot x
\]

why half-densities?

A measure theory lemma: given a commutative
triangle \( X \xrightarrow{T} Y \), and measures \( \mu \) on \( X \)
and \( \nu \) on \( Y \) s.t.

\[
\begin{array}{ccc}
\mu & \xrightarrow{\nu} & \mathcal{E}(
u | q) \\
\downarrow{\mu \xrightarrow{T} \nu} & & \downarrow{\text{seems obvious \& easy}} \\
\end{array}
\]