$M$: oriented 3-manifold, $\xi$: tangent plane field
$\xi = \ker \alpha$, $\alpha \in \Omega^1(M)$

$\xi$ is a contact structure if $\alpha_{\text{ext}}$ is a positive volume form on $M$.

An integral surface for $\xi$.

Problem: classification of contact structures up to isotopy.

**Thm.** (Gray) If $\xi_1, \xi_2$ are close in the $C^1$ sense, then they are isotopic

$\Rightarrow \text{Cont}(M)/\text{isotopy} \text{ is a discrete set.}$

A contact structure is "overtwisted" if

$\exists \hat{\xi} : (M, \hat{\xi})$ s.t. $TD_{2D} = \mathbb{R}^2$

Otherwise it is "tight".

**Thm.** (Eliashberg) $\text{Cont}^+(M)/\text{iso} \cong \frac{\text{Dist}^+(M)}{\text{horotopy}}$

Today $-\Sigma(2, 3, (n-1)) = M = Y_n = \text{surgery on } S^0$

**Thm.** (with Van Horn-Morris)

On $Y_n$ there are $\frac{n(n-1)}{2}$ distinct tight contact structures.
tight contact structures. a triangular number

\[ i = 0 \quad \text{for } Y_i \]

\[ \eta_{ij} \quad 0 \leq i \leq n-2 \quad n-2 \leq j \leq (n-1) \]

\[ \sigma \]

History: 1996: Lisca & Matic distinguished the bottom row using s-w theory
2000: Etnyre-Honda: \( Y \), admits no tight.
2005: Ghiggini: top element is strongly fillable, but not Stein fillable.

My usual question: is there a combinatorial meaning to "tight contact structure"?
Another question: If I had a contact structure, what would I do with it?