From the GPV page of 2006-09:

Given $V$, we need an extension of $V$ to singular semi-virtual knot diagrams, satisfying

$$E_1: \not\not\not \rightarrow \not\not \rightarrow \not\not \rightarrow$$

$$E_2: \not\not \not \rightarrow \not\not \not \rightarrow$$

Failure #1 Extend $V$ by 0. If a diagram has a $\not\not\not$ or a $\not\not\not$, $V$ is zero.

Defect: No reason it should satisfy $E_1$.

Failure #2 Extend $V$ by 0 to $\mathcal{C}_0\not\not\not$

Use $E_1$ to extend it to $\mathcal{C}_0\not\not\not\not\not\not\not$

Defect: No reason it should satisfy $E_2$.

Moral: The extension must take account of the original, best by a “projection” $P: \mathcal{C}_0\not\not\not\not\not\not\not \rightarrow \mathcal{C}_0\not\not\not$

for which $\left( P \circ \mathcal{I} \right)(D)$ is isotopic to $D$, for $\mathcal{C}_0\not\not\not$,

where $\mathcal{I}: \mathcal{C}_0\not\not\not \rightarrow \mathcal{C}_0\not\not\not$ is the obvious inclusion.

Failure #3 Define $P$ by mapping any virtual ring to (say) an under-crossing, extend by means of $E_1$.

Promise: Will satisfy $E_2$?

Defect: Makes no sense! There is no such thing “virtual rings”! And if you force a choice of a PVD representing the given VD, the result will depend on that choice.

Moral: We need an algorithm that given a virtual
knot produces a canonical way of drawing it in the plane, so that when applied to a non-virtually it acts as the identity.

Failure #4: Given a Gauss Diagram, find a spanning tree in it and embed it in the plane, then connect the leaves using descending strands. [Then extend using E1]

Problem: It is hard to understand E2. If a crossing goes virtual, the spanning tree goes bad and we cannot control the difference X - X.

Question: Is this a real defect or just a failure in our ability to prove something?