

From the GPV page of 2008-09:

Given V , we need an extension of V to singular semi-virtual knot diagrams, satisfying

$$E1. \quad \bigcirc = \bigcirc - X$$

$$E2. \quad \underbrace{\bigcirc \bigcirc \dots \bigcirc}_{>n} = 0$$

Failure #1 Extend V by 0. If a diagram has a X or a \bigcirc , V is zero. } more precisely if a diagram has a \bigcirc or is not planar.

Defect: No reason it should satisfy E1.

Failure #2 Extend V by 0 to $CA_0(\mathcal{K})$
use E1 to extend it to $CA_0(\mathcal{K}, \bigcirc)$

Defect: No reason it should satisfy E2.

Moral: The extension must take account of the original, best by a "projection" $P: CA_0(\mathcal{K}, \bigcirc) \rightarrow CA_0(\mathcal{K})$ for which $(P \circ \iota)(D)$ is isotopic to D , for $D \in CA_0(\mathcal{K})$, where $\iota: CA_0(\mathcal{K}) \rightarrow CA_0(\mathcal{K}, \bigcirc)$ is the obvious inclusion.

Failure #3 Define P by mapping any virtual crossing to (say) an undercrossing, extend by means of E1.

Promise Will satisfy E2 \downarrow

Defect: Makes no sense \downarrow . There is no such thing "virtual crossings" \downarrow . And if you force a choice of a PVD representing the given VD, the result will depend on that choice.

Moral We need an algorithm that given a virtual diagram

knot[↖] produces a canonical way of drawing it in the plane, so that when applied to a non-virtual, it acts as the identity.

Failure #4 Given a Gauss Diagram, find a spanning tree in it and embed it in the plane, then connect the leafs using descending strands. [then extend using E1]

Cool fact Hey, there's a particularly lovely way of choosing trees out there!

problem It is hard to understand E2. If a link goes virtual, the spanning tree goes berserk and we cannot control the difference $X - X$.

question Is this a real defect or just a failure in our ability to prove something?