Study the Hartley / Kawauchi theorem about the Alexander polynomial of positive / negative
Amphicheiral knots!

Is there a Kauffman's state model for the Alexander polynomial of virtual / w-knots?

Long framed knot $K \rightarrow$ 2-cable of $K$

cabling

$\rightarrow$ one parameter family of tangles

in the green box

the red arc moves
all under the tangle

to each triple crossing and to
each auto-tangency we associate
the end point of the highest
strand and call it $\tau[p]$
evaluation of the 1-cocycle $\gamma$ on the one parameter family of diagrams gives an element in $H_2 t[1] \oplus H_2 t[2]$. Here, $H_2$ is the Hecke algebra over $\mathbb{Z} [z, z^{-1}, v, v^{-1}]$.

$\gamma(2\text{-cable of } K)$ is an invariant of regular isotopy of $K$. 
Definition of $Y$

Each triple crossing $p$ in the family is replaced by

\[ \rightarrow \langle \overset{\rightarrow}{\rightarrow} \rangle_p - \langle \overset{\rightarrow}{\rightarrow} \rangle_p \]

Here, $\langle \rangle_p$ is the HOMFLYPT polynomial in $H_2$ of the tangle where the triple crossing is replaced by the tangle in the bracket.

In the same way

\[ \rightarrow (v^{-1}-v)/z \langle \overset{\rightarrow}{\rightarrow} \rangle_p \]
\[ Y = \sum_{\text{Def.: all triple crossings } p} \left[ \langle \overrightarrow{\ddag} \rangle_p - \langle \overleftarrow{\ddag} \rangle_p \right] t[p] \]

\[ - \left( \frac{\nu^{-1} - \nu}{z} \right) \sum_{\text{all auto-tangencies } p} \langle \overrightarrow{\ddag} \rangle_p t[p] \]

An example without cabling

\[ Y(\overrightarrow{\ddag}) = Y(\overrightarrow{\ddag \ddag}) + Y(\overrightarrow{\ddag}) \]

\[ = - \frac{\nu^{-1} - \nu}{z} \langle \overrightarrow{\ddag \ddag} \rangle + \langle \overrightarrow{\ddag \ddag} \rangle \]

\[ - \langle \overrightarrow{\ddag} \rangle = -\nu^{-2} \]