The Alek-Tor "divergence"

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Question: What's the natural extension of div to \( \text{Aut}(\mathbb{R}) \)?

\[ U(\text{tor}^n) = \mathbb{R}^n \]

Alek-Tor says it's j:

\[ \frac{d}{ds} j(\exp(su))|_{s=0} = \text{div}(u). \]

\[ \text{div}[a_1, b] = a \text{div} b - b \text{div} a \]

\[ = j(ab) - j(ba) \]

\[ = (j(a) + a j(b)) - (j(b) + b j(a)) \]

I have to remind myself what exactly is the difference between the two exponents in \( \mathbb{R}^n \).

Q: Is \( \text{div}(\mathbb{R}) \) actually isomorphic to \( \text{Aut}(\mathbb{R}) \)?

Ans. Let \( \psi \) be

\[ \psi: (a_1, a_2) \rightarrow (a_1, a_2) \]

\[ \psi \] is YES, but

a choice needs to be made for the direction of the arrows

Q: Is \( U(\text{tor}^n) \) isomorphic to \( \text{Aut}(\mathbb{R}) \)?

Ans.
All roots are at the bottom.

No, roots end up at the bottom of each "layer":

\[ 0 \rightarrow \{ \text{whirls} \} \rightarrow X \xrightarrow{t} \{ \text{twists} \} \rightarrow 0 \]

\[
\text{div} = i^{-1}(t-b) \quad \text{Is the difference of two splittings automatically a 1-cocycle?} \quad \text{Yes:}
\]

\[
(t-b)\left\{ x, y \right\} = [tx, ty] - [b \times by]
\]

\[
= [tx, ty] - [tx, by] + [tx, by] - [b \times by]
\]

\[
= [tx, (t-b)y] + [(t-b)x, by]
\]

\[
= x \text{ div } y + y \text{ div } x
\]
The group case:

\[ 0 \to N \overset{i}{\to} G \overset{\pi}{\longrightarrow} G/N \to 0 \]

Is \( i^{-1}(t(x)^{-1}b(x)) \) automatically a group 1-cocycle? \( i^{-1}(t(x^{-1})b(x)) = \cdot j(x) \)

\[ t(xy)b(xy) = t(y^{-1})t(x^{-1})b(xb(y)) \]
\[ = \overline{t(y^{-1})b(y)b(y^{-1})} t(x^{-1})b(xb(y)) \]
\[ = j(y) + y \cdot j(x) \]