Ober handout corrections

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The Categorification Speculative Paradigm
- Every object in math is the Euler characteristic of a complex.
- Every operation in math lifts to an operation between complexes.
- Every identity in mathematics is true up to homotopy.

The Categorification Tentative Speculative Paradigm
- Every graded algebraic structure in mathematics is the projectivization of a planar ("global") one.
- Every equation written in a graded algebraic structure is an equation for a homomorphic expansion or, for an automorphism of such.

Graded Equations Examples
- The Aleskevich-Torossian equations
  \( \Phi = \Phi_{\phi} = (F^{12})^{-1} (F^{13})^{-1} F^{12} F^{13} \in H(\delta_{13}) \)
- The pentagon: \( \Phi = 12^I \Phi_{\phi} \Phi_{\phi} = 12^I \Phi_{\phi} \Phi_{\phi} \)
- The Aleskevich-Torossian equations
  \( F \in H(\delta_{14}) \)
  \( \Phi = \Phi_{\phi} = (F^{12})^{-1} (F^{13})^{-1} F^{12} F^{13} \in H(\delta_{14}) \)

Defining proj \( O \)
- The augmentation "ideal":
  \( I \) = \( \theta = \{ \text{formal differences of objects of the same kind} \} \)

Theorem: \( \Phi \) is "type n invariant".
- \( \{ F \}^{(n+1)} \) is "weight systems".
- proj \( O \) is \( \mathbb{A} \), "chord diagrams".

Warmup Examples:
- The projectivization of a group is a graded associative algebra.
- A quandle: a set \( Q \) with a binary op \( \wedge \) s.t.
  \( 1 \wedge x = 1, z \wedge 1 = x \wedge x = x, (\text{apples}) \)
  \( z \wedge (y \wedge z) = (x \wedge z) \wedge (y \wedge z) \)
- proj \( Q \) is a graded Lie algebra: set \( \hat{v} := e^{-v}1 \) (these generate \( F \)), feed 1, 1, 1, \( y, 1 \) in \( \mathbb{K} \), and collect the surviving terms of lowest degree.

An Expansion is \( Z: - \longrightarrow \text{proj} O \) s.t. \( Z(F)^n \subset \text{proj}(O)^n \subset \text{proj} O \). A "universal finite type invariant".

For an expansion \( Z: - \longrightarrow \text{proj} O \) with \( \theta = \theta_{\text{proj} O} \), then \( \mathbb{K} \) and \( F^{(n+1)} \) p...