Localization of Circle Actions

March-05-08
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$a - b\pi = 0$ Yael Karshon, Nov 2007.

Localizing a v.f. ξ action on a manifold M:

If $d_\xi \alpha = 0$ then $\int_M \alpha = \sum_{p \in M^\xi} \frac{\alpha(p)}{\sqrt{\det d\xi}}$ ← Product of weights of the action of ξ on T_pM.

where $d_\xi := d + \iota_\xi$

<u>Proof</u> Find $\Theta \in \Omega^1(M - M^\xi)$ s.t. $\Theta(\xi) = 1$, and then (using Leibnitz)

$$d_\xi\left(\alpha \wedge \Theta \wedge (d_\xi \Theta)^{-1}\right) = \alpha \wedge (d_\xi \Theta) \wedge (d_\xi \Theta)^{-1} = \alpha$$

where $d_\xi \Theta = 1 + d\Theta$ and so $(d_\xi \Theta)^{-1}$ makes sense.

Finally, near a fixed point,

$$\Theta = \frac{\sum \frac{1}{\eta_j} r_j^2 d\theta_j}{\sum r_j^2} \quad \text{in coords } (r_j, \theta_j)_{j=1}^n.$$